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Electrodiffusional probe for measurement of the wall shear rate vector

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1. INTRODUCTION

The measurement of the wall shear rate magnitude by means of the limiting electrodiffusional current is a well-known technique [1–3]. In order to measure not only the magnitude but also the direction of the wall shear rate vector, rectangular or circular probes, composed of two parts separated by a thin insulation gap, have been used [4, 5]. From the sum and the difference of the currents measured on each part of these bi-electrodes it is possible to deduce the two components of this vector parallel to the wall. However, with this kind of bi-electrode, the determination of the direction θ of the wall shear rate is obtained from $\sin\theta$, and so the precision of the θ value is good only for small enough θ. This problem of precision disappears with the square probe, composed of four small square electrodes, proposed by Wichterle and Zak [6]. However, in this case, the signal treatment is very difficult as the total current depends on the flow direction. Recently, Wein and Sobolik [7] proposed a circular probe, composed of three sectors, for this kind of measurement. With this geometry, the precision is independent of θ values and the total current is independent of the flow direction. Now, by means of a new technique [8] it is possible to manufacture such a probe. Because of geometrical imperfections, this probe needs some correction factors which must be determined by experimental calibration.

The theory of the multiple-sector circular probes, and practical indications for treatment of current data in order to get the flow direction, are given in the first part of this paper. In the second part, the results of the calibration of a probe, manufactured by means of the new technique mentioned above, are presented and discussed. This calibration has been made in the coaxial cylinders and in the cone-plate devices.

2. THEORY OF MULTIPLE CIRCULAR-SECTOR PROBES

If a probe is composed of sectors separated by a thin insulation gap, then the electric current through each sector depends on its position relative to the flow direction. The current through the front sector is greater than the current through the rear sector lying in the wake of the concentration boundary layer. The current density on a probe flush mounted in the wall depends on the distance from the front edge of the probe

$$J = Kx^{-1/3} \tag{1}$$

where $K = 0.5384 nFc_0 D^{2/3}\gamma^{1/3}$. This equation is obtained from Lévêque's solution and Faraday's law.

The electric current through a sector or through the whole probe can be calculated by integration of equation (1) over the sector or the probe surface. For a circular probe this equation becomes

$$i = 2.15695 nFc_0 D^{2/3} R^{5/3} \gamma^{1/3}. \tag{2}$$

The normalized current ($F(\beta) = i_s/i_a$) through any individual sector can be easily derived [7]. For a convex sector ($0 < \beta \leqslant \pi$) orientated with one edge parallel to the flow direction, the normalized current depends only on the position of the other edge of the sector (see Fig. 1)

$$F(\beta) = 0.374S\int_0^1 (W-Ct)^{2/3}\,dt \quad \text{for } 0 < \beta \leqslant \pi/2 \tag{3a}$$

or

$$F(\beta) = 0.5-0.374S\int_0^1 [(2W)^{2/3}-(W-Ct)^{2/3}]\,dt \quad \text{for } \pi/2 < \beta \leqslant \pi \tag{3b}$$

where $S = \sin\beta$, $C = \cos\beta$ and $W = (1-S^2t^2)^{1/2}$.

For a convex sector which has an arbitrary orientation with respect to the flow direction, the normalized current can be obtained as an algebraic combination (sum and/or difference) of the normalized currents over the whole probe ($F(\beta) = 1$) and those over other sectors for which equations (3a) and/or (3b) are valid. In general, four different situations, according to β_1 and β_2 values (see Fig. 2), can be identified with the relevant values of the normalized currents listed in Table 1.

The normalized current through a sector, characterized by the angle 2α and orientated by the angle ϕ between its bisector and the flow direction (see Fig. 2), can be expressed by a Fourier series [7]

$$I_s(\alpha,\phi) = i_s/i_a = \alpha/\pi + \sum_{m=1}^{\infty} C_m \sin(m\alpha)\cos(m\phi). \tag{4}$$

The C_m values are given in Table 2.

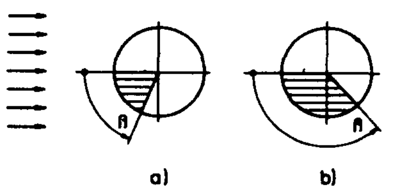

Fig. 1. Sector with one edge parallel to the flow direction. (a) $0 < \beta \leqslant \pi/2$; (b) $\pi/2 < \beta \leqslant \pi$.

NOMENCLATURE

a_{ij}	Fourier coefficients in equation (5)
b_{ij}	Fourier coefficients in equation (5)
c_0	bulk concentration of species [mol cm^{-3}]
C_m	Fourier coefficients of ideal three-sector probe
D	diffusion coefficients of species [cm^2 s^{-1}]
F	Faraday's constant, 96 487 C equiv^{-1}
$F(\beta)$	normalized current through a sector orientated with one edge parallel to the flow direction
I_s	normalized current as a function of α and ϕ
I_i	normalized current through the ith sector
i_s	current through the circular probe [A]
i_s	current through a sector [A]
J	current density [A cm^{-2}]
n	number of electrons per ion reaction
R	probe radius [cm]
x	distance measured from the front of the probe [cm].

Greek symbols

α	half angle of a sector (see Fig. 2) [rad]
β, β_1, β_2	angles defined in Figs. 1 and 2 [rad]
ε_i	angle between the ith bisector and the reference direction
θ	flow direction measured with respect to the reference direction
ϕ_i	angle between ith bisector and the flow direction.

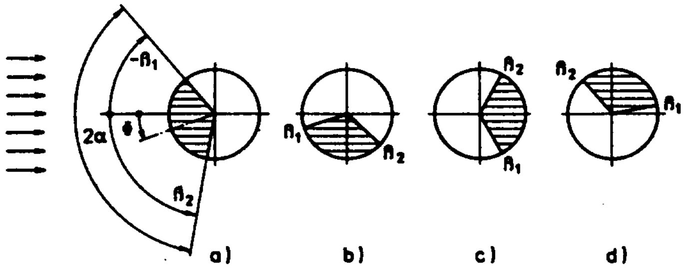

FIG. 2. Sector orientation for normalized current calculation as function of β_1 and β_2. For (a), (b), (c) and (d), see Table 1.

For practical measurements, it is necessary to define, with a mark on the body of the probe, an arbitrary reference direction ($\theta = 0$) from which the flow direction angle θ is measured. In the general case, where the electrode is divided into 'n' sectors, this angle θ is related to the angle ϕ by the expression

$$\theta = \phi_i - \varepsilon_i$$

where i stands for any sector ($1 \leqslant i \leqslant n$) and ε_i is the angle between the bisector of the ith sector and the reference mark. Therefore, expressing ϕ as a function of θ in equation (4) leads to a Fourier series involving $\cos(j\theta)$ and $\sin(j\theta)$ as well. In this series each sine term stands for the angle shift ε_i of the ith sector. Thus, the normalized current through the ith sector as a function of θ can be expressed by

$$I_i(\theta) = I_s(\alpha_i, \phi_i) = \alpha_i/\pi + \sum_{j=1}^{\infty} a_{ij} \sin(j\theta) + \sum_{j=1}^{\infty} b_{ij} \cos(j\theta). \quad (5)$$

Table 1. Normalized current of an arbitrary orientated sector as a function of β_1 and β_2 (see Fig. 2)

Case	Angle values	Normalized current
(a)	$-\pi < \beta_1 < 0 < \beta_2 < \pi$	$F(\beta_2) + F(-\beta_1)$
(b)	$0 < \beta_1 < \beta_2 < \pi$	$F(\beta_2) - F(\beta_1)$
(c)	$0 < \beta_2 < \pi < \beta_2 < 2\pi$	$1 - F(2\pi - \beta_2) - F(\beta_1)$
(d)	$\pi < \beta_1 < \beta_2 < 2\pi$	$-F(2\pi - \beta_2) + F(2\pi - \beta_1)$

The dependence of the normalized currents of the sectors on the flow direction (directional characteristics) for an ideal circular probe, which consists of three identical sectors, $\alpha_i = \pi/3$ ($1 \leqslant i \leqslant 3$), is shown in Fig. 3. For practical reasons it is almost impossible to manufacture a geometrically perfect circular probe with three identical sectors. Therefore, the directional characteristics of a real probe (see micrograph in Fig. 4) differ from those of an ideal one and must be estimated experimentally. There are two ways to get the flow direction θ with respect to the reference direction.

The first one, which is the most quantitative, requires experimental measurement of normalized currents for some θ values and hence deduction of the Fourier coefficients α_i, a_{ij} and b_{ij} relevant to a specific sectored probe. θ values may then be obtained from the values of normalized currents by solving equation (5) for at least two sectors to get an unambiguous determination.

The second way makes use of the fact that the directional characteristics between two successive cross-points can be fairly well approximated by six straight lines (see Fig. 3). The

Table 2. Fourier coefficients of an ideal multiple sectors probe

m	1	2	3	4	5	6
C_m	0.12614	0.00702	-0.00303	-0.00064	0.00051	0.00015

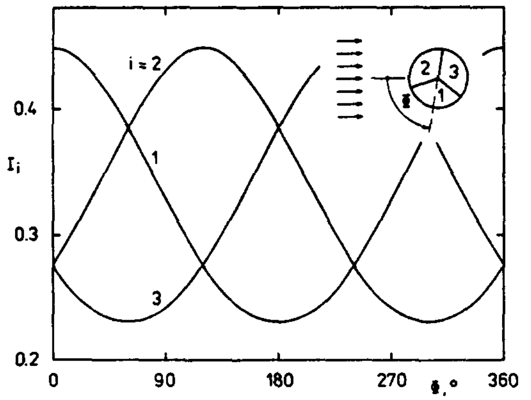

FIG. 3. Directional characteristics of an ideal three-sector circular probe.

theoretical values of the normalized current at any cross-points are equal either to 0.384 or to 0.276. In this way, the calibration results in six linear equations corresponding to six different domains, obtained by permutation of the relative values of the normalized currents (I_1, I_2, I_3). By comparison of these relative values (e.g. $I_3 < I_1 < I_2$ in Fig. 3), the pertinent normalized current (e.g. I_1) and the corresponding linear equation can be found, and then solved for θ. This numerical procedure is very fast.

3. EXPERIMENTAL

A three-sector probe was manufactured by means of a new technique [8]. The originally circular cross-section of three 0.5 mm diameter platinum wires was transformed into a sector cross-section [8]. The wires were then glued together with epoxy resin poured in a stainless steel tube which served as the counter electrode. A micrograph of the probe is shown in Fig. 4.

The probe was calibrated in both a cone-plate device and a coaxial cylinders device. The cone, of 26 mm diameter, was made of organic glass. The probe was flush mounted in a stainless steel disk of 40 mm diameter, which served as the plate. The angle between the cone and the plate was 2°. The other apparatus consisted of two coaxial cylinders of 293 and 303 mm diameters, respectively. The probe was flush mounted in the wall of the outer cylinder. In both devices it was possible to turn the probe around its axis and so adjust the relative flow direction.

The test fluid was a 0.025 mol l^{-1} equimolar potassium ferro/ferricyanide solution with 80% by weight of glycerol and 10 g l^{-1} of K_2SO_4. The viscosity of the solution was 6.25×10^{-5} m^2 s^{-1} at 20°C. The potential applied on all three segments was -0.7 V, corresponding to the diffusion limiting current of the potassium ferricyanide reduction.

4. RESULTS AND DISCUSSION

The dependence of the total current on the wall shear rate, shown in Fig. 5, is a straight line in log–log coordinates with

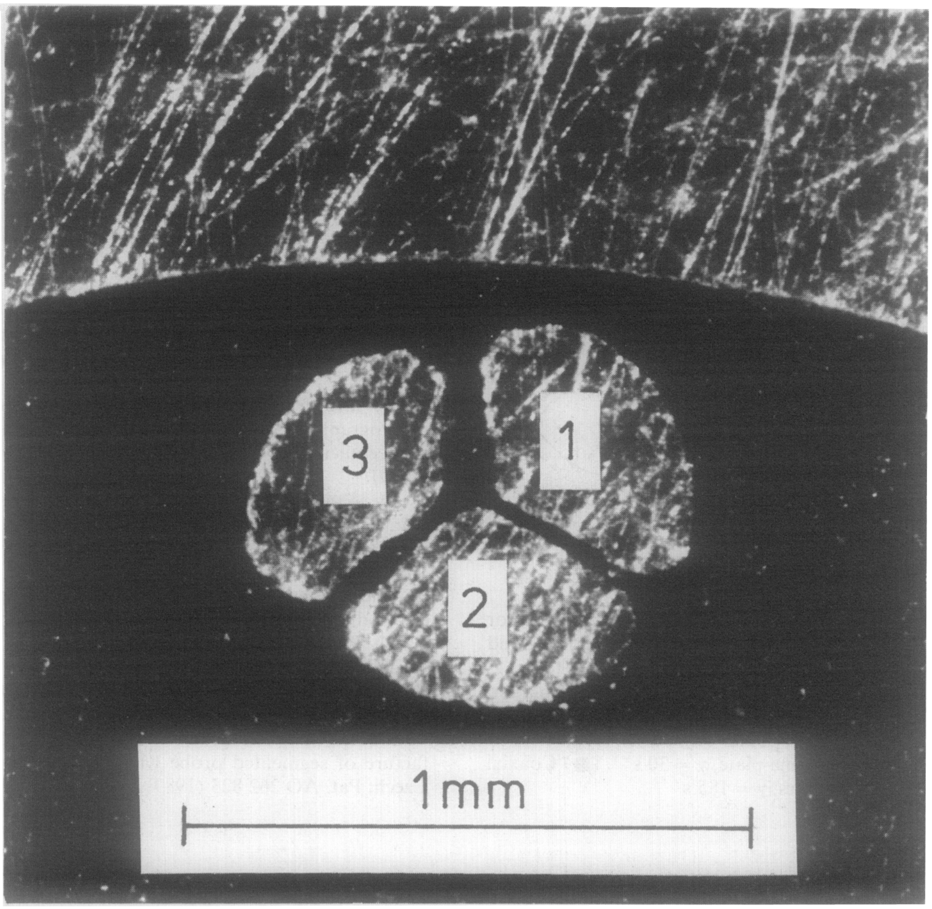

FIG. 4. Micrograph of the probe. Sectors are denoted by numbers. The light area at the top is a part of the counter electrode made of stainless steel and the dark area is the epoxy resin insulation.

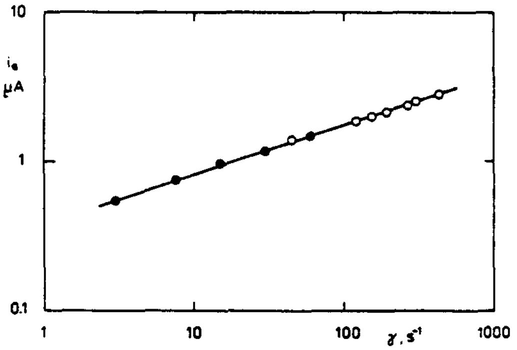

FIG. 5. Total current as a function of the wall shear rate. (○) Coaxial cylinders. (●) Cone-plate.

a slope very close to the theoretical value of 1/3 (see equation (2)). The standard deviation of the total currents measured at a constant shear rate but for different flow directions is only 0.7%.

For the directional characteristics of the probe, shown in Fig. 6, there is good agreement between the measurements made with different devices: cone-plate and coaxial cylinders. The small deviations, which are probably caused by geometrical imperfections of these devices, are random. Thus, it is possible to conclude that the theoretical result concerning the independence of directional characteristics on the total current and in particular on the magnitude of shear rate vector (see equation (4)) is proved experimentally. The solid lines (Fig. 6) stand for the Fourier series (5) calculated from measured data and including the $j\theta$ terms up to $j = 2$. The normalized current of the second sector has the greatest standard deviation from the solid line (0.009). If the terms up to $j = 4$ are included in the Fourier series, this standard deviation reduces to 0.0016.

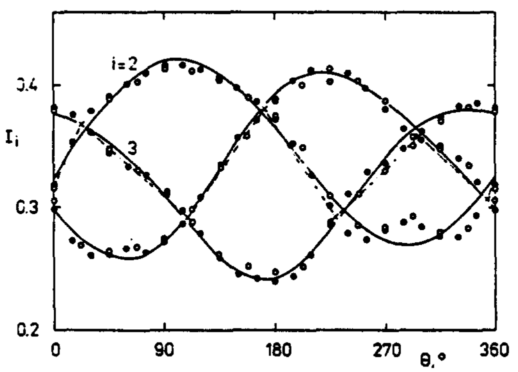

FIG. 6. Directional characteristics of the real three-sector circular probe determined by experimental calibration. Solid lines stand for Fourier series including $j\theta$ terms up to $j = 2$. Dashed lines represent the six straight lines for calculation of the flow direction by linear approximation. These lines are delimited by the cross-points of Fourier series with $j\theta$ terms up to $j = 4$. (○) Cone-plate, $\gamma = 30\ s^{-1}$. (●) Coaxial cylinders, $\gamma = 185\ s^{-1}$.

By means of this Fourier series, the cross-points denoted by cross-symbols were found. These are the boundary points of the straight lines which can be used for the determination of the flow direction θ in the linear approximation method defined above. The standard deviation of the angles estimated by this procedure is 5°. There are two unexpected secondary maxima on the measured normalized currents: one for I_1 and the other for I_2 corresponding respectively to θ of about 60° and 300°. These deviations are caused by the wide gaps between sectors: for these two flow directions, new concentration boundary layers start on the rear sector because there is no mass transfer on the insulation gap between the two front sectors.

5. CONCLUSION

The three-sector circular electrode has the most convenient geometry for measuring the wall shear rate vector: magnitude and direction. Contrary to the two-sector circular electrode, it gives good and constant precision for all flow directions. Compared to the square four-segment probe, the numerical determination of the flow direction is very fast.

Due to geometrical imperfections of the three circular sectors, inherent in its manufacture, directional characteristics are a property of each probe and must be obtained experimentally. However, the calibration curve for a probe can be used for all electrochemical reactions and fluid flows, because the directional characteristics depend neither on the total current nor (in particular) on the magnitude of the shear rate vector.

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